




## Article

# The Apse of the Gothic Cathedral of Tortosa versus Augustine of Hippo's Civitate Dei

Cinta Lluis-Teruel , Josep Lluis i Ginovart, Iñigo Ugalde-Blázquez  and Ricardo Gómez-Val \* 

Department of Architecture, Universitat Internacional de Catalunya, 08017 Barcelona, Spain; cintalluis@uic.es (C.L.-T.); jlluis@uic.es (J.L.i.G.); iugalde@uic.es (I.U.-B.)

\* Correspondence: rjgomez@uic.es

**Abstract:** This research delves into the influence of St. Augustine on the construction of the Gothic cathedral of Tortosa. The canonical cathedral of Tortosa underwent re-establishment in 1155, which was carried out by Bishop Godfrey who was the abbot of Saint Rufus of Avignon and was governed by *Beati Augustini* rule. The presence of St. Augustine in the Capitular archives with *De Civitate Dei* (ACTo-20) from the XII century is examined. This, coupled with a spatial analysis of the liturgical space using laser scanning (TLS), serves to validate the historiographical thesis put forth by Wilhelm Worringer, Erwin Panofsky, and Otto von Simson for understanding the construction of the apse of the Gothic cathedral (1346–1441). This methodology establishes a bijection between patristic and Neoplatonic sources and the interpretation of the liturgical space's dimensions using statistical systems. This approach addresses the construction of the apse through the incorporation of a heptagon, a geometric figure that is absent in Euclid's *Elementa* and Ptolemy's *Almagest*. In conclusion, it is determined that both the imagery and metrics employed in the design of a radial heptagonal apse, as well as its cross-section, are influenced by both St. Augustine and the metrics of the *Neoplatonics*, which remain present in the Chapter Archives.

**Keywords:** Augustine of Hippo; cathedrals; Neoplatonism; proportion; sacred geometry



**Citation:** Lluis-Teruel, Cinta, Josep Lluis i Ginovart, Iñigo Ugalde-Blázquez, and Ricardo Gómez-Val. 2024. The Apse of the Gothic Cathedral of Tortosa versus Augustine of Hippo's Civitate Dei. *Religions* 15: 943. <https://doi.org/10.3390/rel15080943>

Academic Editors: Alba Arboix-Alió and Magda Maria Serrano

Received: 22 April 2024

Revised: 23 June 2024

Accepted: 22 July 2024

Published: 5 August 2024



**Copyright:** © 2024 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

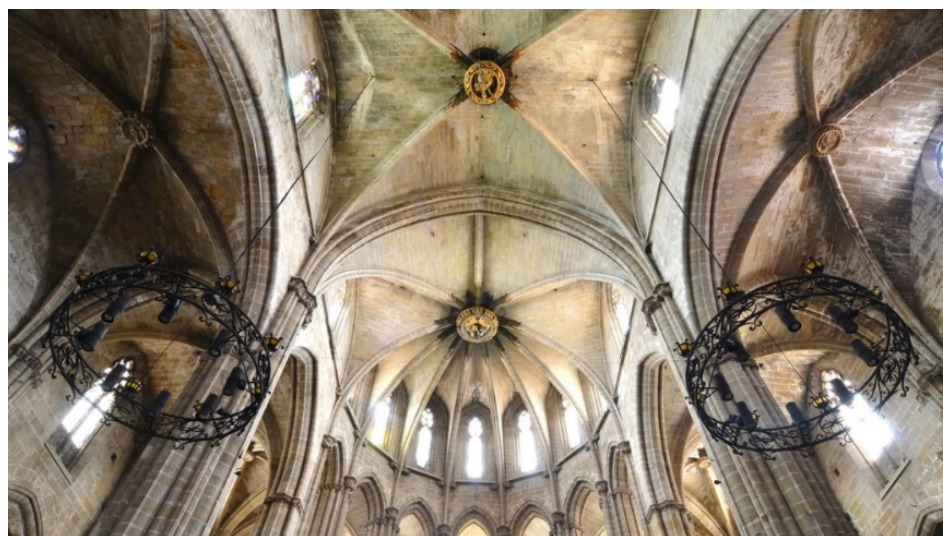
## 1. Introduction and Objective

The canon of Tortosa was refounded in 1155 by Bishop Godfrey (+1165), abbot of San Rufo de Avignon, following the rule 'vivere sub regula Beati Augustini, et iuxta consuetudines Ecclesiae Sancti Ruffi' (García 1998, pp. 10–13). Regarding la regula, it originally dates back to 1039 when Benedict, Bishop of Avignon, oversaw the functions of the choir and the recitation of the Psalter (Misonne 1963, pp. 471–89). It was governed similarly to other cathedrals in the Catalan territory, such as Vic, Lleida, Girona, and Seo de Urgel (Calvo 2014, pp. 78–79), where the request for books was already ordained within the canonical schedule (Rule 37) (Orozco 1881, p. 36). From this moment on, the construction of the Romanesque cathedral began in 1158, and it was consecrated on 28 November 1178. On 20 April 1346, Bishop Arnau de Llordat (1341–1346) and the Chapter commissioned the *magister operis*, Bernat Dalguaire (+1347), a new cathedral that was to replace the *ecclesia vetulam*. (ACTo, NC 1346. 11).

Indirect measurement procedures such as close-range photogrammetry (CRP) and terrestrial laser scanning (TLS) allow for the analysis of the construction reality of the Tortosa Cathedral. This makes it possible to carry out graphic experiments on architectural objects and obtain contrasted explanations relative to the hypotheses based on the geometric and constructive research of seo dertosense<sup>1</sup>.

The results obtained from these records can be statistically analyzed. These numerical values are subjected to explanatory treatment, attempting to determine the causes and consequences of the construction of the sacred space of the cathedral. The evolution of formal changes is considered a concrete phenomenon established by liturgical and

technical changes during the gestation and construction period of the Gothic apse (1346–1441) (Figure 1).



**Figure 1.** The apse of the Cathedral of Santa María de Tortosa (1346–1441). Presbytery keystone with the Coronation of the Virgin and Saint Augustine.

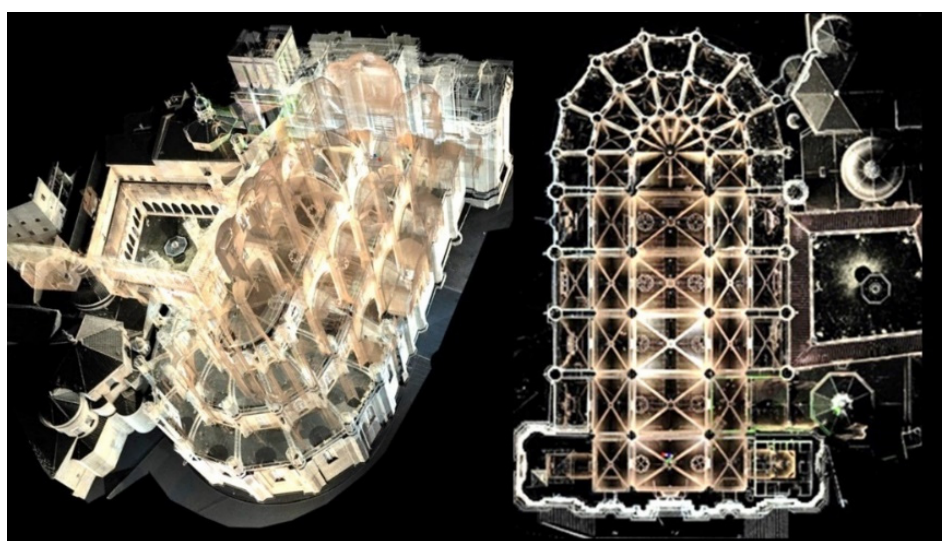
Not only the *what* but also the *why* of the causes is sought; therefore, the evolution of this liturgical space and how it has reached its current state are studied. The objective is to create an explanatory model that allows for the observation of the sequences of cause and effect among the promoters, the Chapter and bishop, and the magister operis who have worked on the Tortosa Cathedral. In this way, it is explained, from the perspective of complexity, that this sacred space has been generated by the complementation of both types of knowledge, which were defined as *scientia theoria* and *scientia practica* from the perspective of medieval philosophy. This concept spread in Gothic Europe from Domingo de Gundisalvo's *De Scientiis* (fl. 1150) and its precedent, al-Fārābī's "Catalog of the Sciences" (c. 870–950).

Al-Fārābī states that *Scientia doctrinali* includes arithmetic, geometry, optics, astronomy, mathematics, music, the science of weights, and engineering; it specifies the difference between theory and practice (González 1932, pp. 97–105). Gundisalvo uses these same terms in the *De Scientiis* (Alonso 1955, pp. 85–112), definitions disseminated to the world of European cathedrals by Vincent of Beauvais (c. 1194–1264) in the *Speculum Doctrinale* (Mâle 1910, pp. 37–40).

Classical historiography assumes in the scholastic context that the Gothic cathedral is the manifest result of the will of two parties: that of the promoter and that of the builder. The interaction between both figures was highlighted by Wilhelm Worringer (1881–1965) in the *Formprobleme der Gotik* (Worringer 1911) and by Erwin Panofsky (1892–1968) in *Gothic Architecture and Scholasticism* (Panofsky 1951). Otto von Simson (1912–1993) initially tackled the question of learned sources (Simson 1952, pp. 6–16), seeking them in Augustine of Hippo's *De Civitate Dei*, *De Ordine*, and *Musica*, and further developed them in *The Gothic Cathedral: The Origins of Gothic Architecture and the Medieval Concept of Order* (Simson 1956). Methods utilizing indirect records such as close-range photogrammetry (CRP) and terrestrial laser scanning (TLS) (Figure 2) allow for a precise analysis of the three-dimensional space, enabling the examination of points that were previously inaccessible and allowing them to be contrasted with these historiographical theses.

Saint Augustine acknowledges in the *De ordine, libri duo* (386) that the Trivium and Quadrivium programs were the basic instruments for understanding the Holy Scriptures. He proposes the formation of man in the arts (De Ord. II.16, 44)<sup>2</sup> as they promote the elevation of the spirit on the path toward God. He considers that moderate and rational erudition in the

liberal arts makes man more agile and steadfast in encountering the truth of his knowledge<sup>3</sup>. In the *Confessionum, libri tredecim* (397), geometry, music, and arithmetic are found, because the ability to understand and the acuity in discerning are gifts from God (Conf. IV, 16, 30)<sup>4</sup>. He reports that what unfolds reasonably in ordered forms was designated by the name rhythm, which in Latin can only be called numerus (De Ord. II, 14, 41)<sup>5</sup>. Thus, he sees the need for the study of numbers for the order of music, geometry, and the movement of the stars (De ord. II, 5, 14)<sup>6</sup>. In the *De Genesi ad Litteram libri duodecim* (401), he recalls that the number without number is that by which all things are formed. The weight without weight is that by which balance is established, reducing all things to stillness (De Gen. ad litt. IV.8)<sup>7</sup>. He defines geometry in the beauty of figures, in the figures and dimensions, and in the dimensions and numbers. He also inquires whether lines and spheres or any other form and figure in reality exist as they are contained in the intellect, calling geometry the science that distinguishes and orders these understandings (De Ord. II.15, 42)<sup>8</sup>.



**Figure 2.** A terrestrial laser scan (TLS) of the Tortosa Cathedral.

With these data, the objective is to explore the influence of the thought of Augustine of Hippo (354–430) through the Augustinian Chapter on the established order in the construction of the cathedral, using as reference the codices of the Capitular Archive of Tortosa (ACTo). Saint Augustine himself states in *De libero arbitrio libri tres* (388–391) that we all desire to be happy and wise, for no one can be happy without being wise (lib. arb. II, 9, 26)<sup>9</sup>.

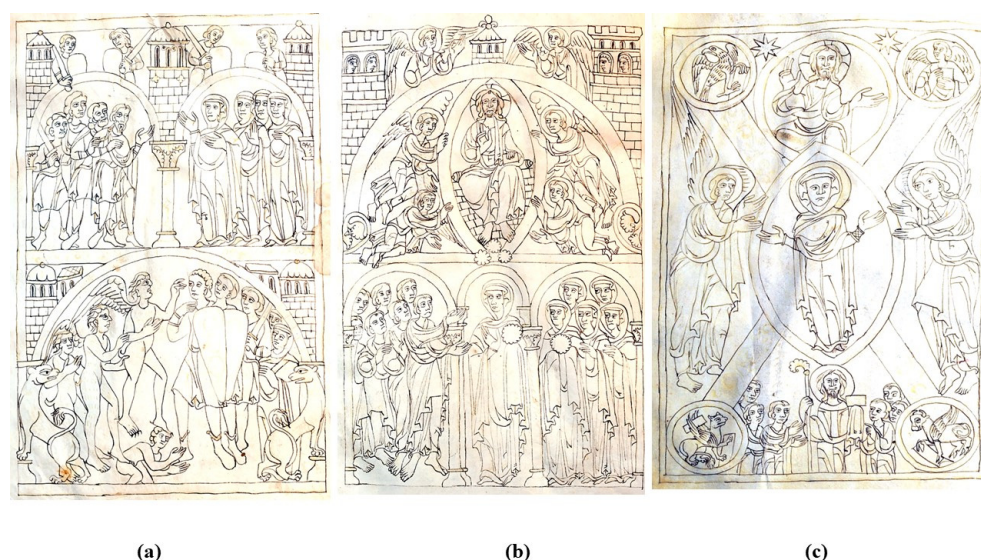
The Augustinian model inherited by monks distinguishes between *uti* and *frui*<sup>10</sup>, relating to the use of temporal realities, wisdom, and the enjoyment of the eternal. These terms also applied to the right of usufruct, which can be indirectly applied to the construction of cathedrals. At the threshold of the Renaissance, the Christianized Neoplatonic model is subtly linked to the sensible and rational dimensions of conjectural science. Thus, the canon of Noyon and mathematician, Charles Bovelles (1483–1553), completes this journey by revisiting Saint Augustine to show the Trinitarian dimension of the overcoming of practical or erudite knowledge (Trottmann 2015, pp. 805–16). In this way, the order of Gothic construction allows us to address geometric and proportional issues that stem from the knowledge concentrated in some cases in Augustinian works.

An example is how the heptagonal apse of the Tortosa Cathedral was drawn. Charles Bovelles himself acknowledged in the *Livre singulier et utile, touchant l'art pratique de geometrie* (1542) that a figure as important for Christian symbolism as the heptagon did not appear in Euclid's *Elements* (c. 325–c. 265 BC) (Bovelles 1542, 25v–28r). The French humanist had already discussed and constructed this figure in the *Geometricum Introductorium* (1503), published in Paris in 1510 (Bovelles 1510, p. 196), a work present in Tortosa (ACTo 300) (Figure 3)<sup>11</sup>.



(1875–1952) found four illustrations (fol. 1r–2v) (March 1916, pp. 351–54). Later, Jordi Rubió i Balaguer (1887–1982) and Ramon d’Alòs-Moner i de Dou (1885–1939) added the initial folios of the codex and the illustration (fol. 5r) (Rubió and Rubió 1914, p. 143).

In the current catalog, it is listed as ACTo 20, Saint Augustine, Bishop of Hippo: Of the City of God. It is a manuscript from the 12th century written on parchment, with 408 folios measuring 370 × 270 mm, and it is placed in a writing box with two columns measuring 280 × 210 mm with 33 lines (fol. 13r). It lacks part of the first book (Bayerrri 1962, pp. 157–59). It contains five illustrations cited as follows: the Zodiac (fol. 1r), the Defense of the City of God (fol. 1v) (Figure 4a), the Celestial Jerusalem (fol. 2r) (Figure 4b), the *Maiestas Mariae* (fol. 2v) (Figure 4c), and the Creation (fol. 5r) (Ibarburu 1984, pp. 93–124). Additionally, it contains 21 initials that head each of the books except for XVIII (Ibarburu 1985, pp. 103–25). In the review through *Les manuscrits à peintures de la Cité de Dieu de Saint-Augustin* by Alexandre Laborde (1853–1944) with 61 illuminated codices, we can relate the Zodiac (fol. 1r) with the reference from Mss. Franc. 900S-9006, fol. 287v, which is from the Royal Library of Brussels (c. 1410), illustrating that Book VIII is dedicated to natural theology (Laborde 1909, p. 322).

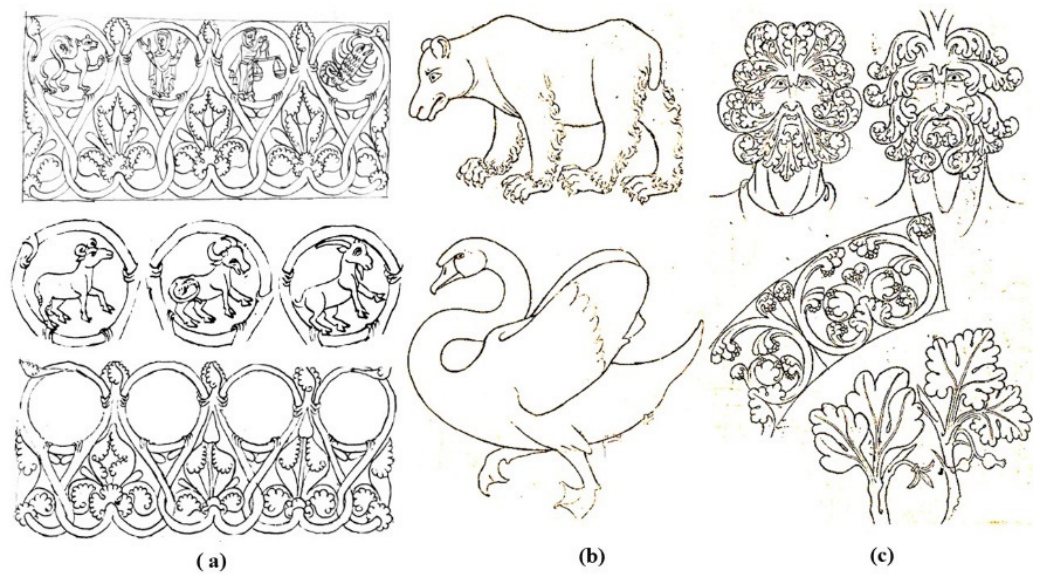


**Figure 4.** ACTo No. 20, 12th century. (a) fol. 1v, Defense of the City of God; (b) fol. 2r, Celestial Jerusalem; (c) fol. 2v, *Maiestas Mariae*. Image authors granted reproduction permission.

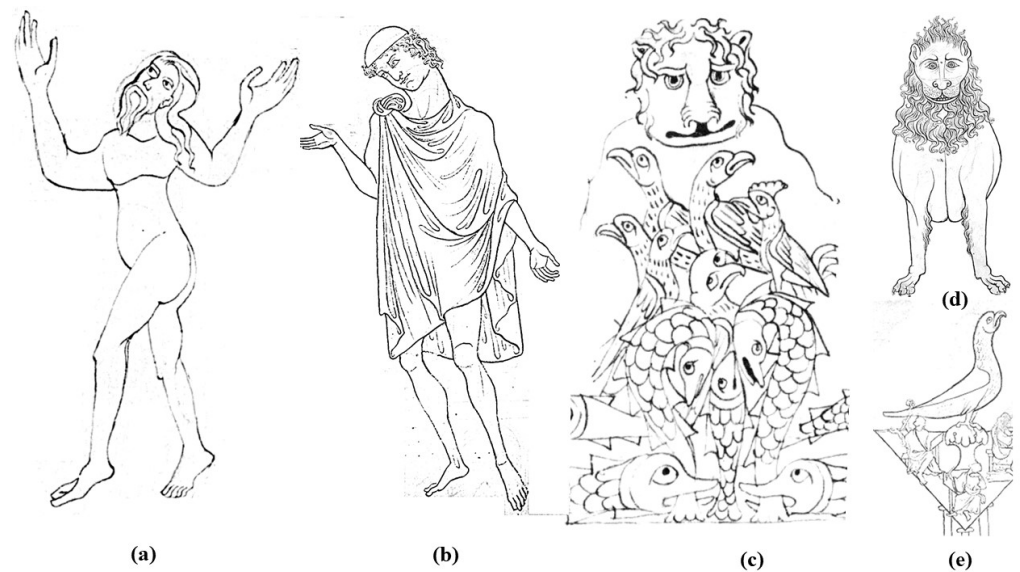
The Defense of the City of God (fol. 1v) (Figure 4a) bears some resemblance to the illustrations of Book I, depicting the sack of Rome by the Goths and showing how God’s mercy tempered the destruction of the City (BnP, Ms. Franc. 22912-13) (c. 1376) and (Ms. Add. 1 5244-45) from the British Museum (1370–1377) (Laborde 1909, p. 191)). The image of Celestial Jerusalem (fol. 2r) (Figure 4b) resembles that of the Heavenly City from Ms. lat. A. 7, fol. 1v of the Metropolitan Chapter Library of Prague’s illustration of Book I (Laborde 1909, Plate IV). The *Maiestas Mariae* (fol. 2v) (Figure 4c), for which we have no direct reference in the cataloging and which is long considered an Assumption, can be read as an allegory of the Church by virtue of the hierarchical arrangement of Christ, the Virgin, and below them, a figure with a pastoral staff and a book, perhaps St. Augustine, is surrounded by faithful and disciples (Grimaldi 2015, pp. 77–90). The Creation (fol. 5r) could illustrate Chapter 21 of the Creation of the first man and the human race from Book XII, reflected in a large number of codices (Laborde 1909, pp. 193–95).

On the other hand, the architectural layout of the illustrations of the City of God and Celestial Jerusalem (Figure 4a,b) brings us closer to the collection of graphic representations of architecture in the *Livre de portraiture* (c. 1220–1240) by Villard de Honnecourt (c. 1200–c. 1250) from (BnP, Ms Fr 19093), which consists of 33 parchment folios (0.230 × 0.240)<sup>12</sup>.

Likewise, the vegetal forms and the bestiary depicted in the Zodiac (ACTo 20 fol. 1r) in the representations of Aries, Taurus, Leo, or Capricorn (Figure 5a) bring us closer to the bear and the swan, a figurative representation of Celestial Jerusalem from (BnP, Ms Fr 19093, fol. 4r) (Figure 5b). The *têtes de feuilles* and the vegetal ornaments (BnP, Ms Fr 19093, fol. 5v) (Figure 5c) bring us closer to the scrolls alternating with foliar motifs, which are symmetrical and heart-shaped motifs from (fol. 1r). The human figures of the Creation of Adam and Eve (Figure 6a) bring us closer to those in (BnP, Ms Fr 19093, fol. 29v) (Figure 6b), an ancient representation of Mercury; to the birds and marine animals of the creation of the fifth day and the terrestrial ones of the sixth day (Figure 6c); and to the lion (BnP, Ms Fr 19093, fol. 24v) (Figure 6d) and eagle for reading the gospel (BnP, Ms Fr 19093, fol. 7r) (Figure 6e).



**Figure 5.** ACT 20: (a) representation of Zodiac. BnP, Ms Fr 19093, (b) “les têtes de feuilles”; (c) vegetal ornaments<sup>13</sup>.



**Figure 6.** Human figures: (a) ACTo 20; (b) BnP, Ms Fr 19093. Marine and terrestrial animal figures: (c) ACTo 20; (d) BnP, Ms Fr 19093<sup>14</sup>; (e) BnP, Ms Fr 19093, fol. 7r.

### 3. Methodology and Results of Terrestrial Laser Scanner (TLS)

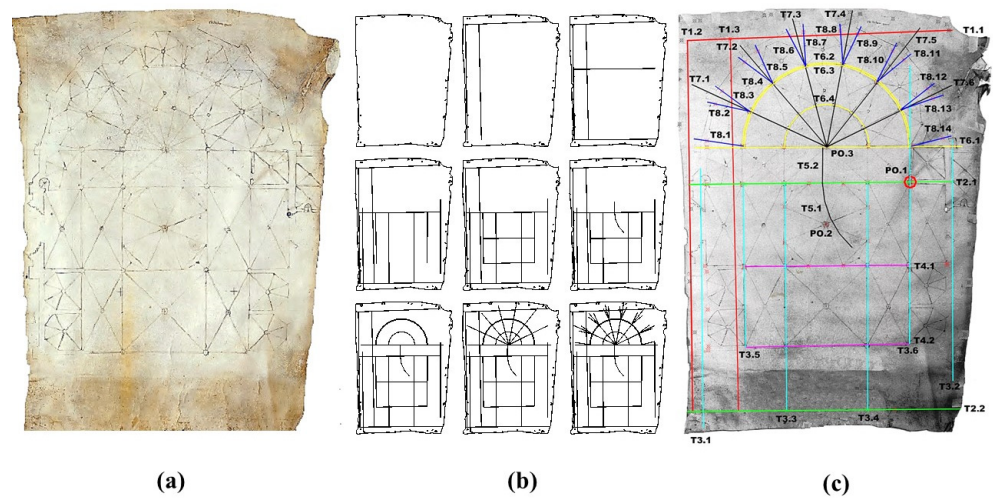
The basic pattern of measurements appearing in the Llibres d'Obra [ACTo] is the *cana* of eight spans and the span of twelve fingers. The *cana* of Tortosa is defined in Book IX, No. 15.5 of the Consuetudines Dertosae (1272) [AHCTE: cod. 53, fol. 256r]<sup>15</sup>. By comparing the unifying documents of the *cana* of Tortosa with that of Barcelona (24-VII-1593), we concluded that the *cana* of Tortosa used for the cathedral measures 1.858 m, and the span is 0.2323 m. The methodology proposes a comparative reading of these measurements with the data obtained through a survey conducted using a Leica Scan Station C10 with an accuracy of 4 mm at 50 m and angular accuracy of 12"/12" (Figure 7).



**Figure 7.** An image resulting from the terrestrial laser scanner (TLS) survey of the apse of the Tortosa Cathedral.

The total error of the methodological process ( $E_t$ ) is the sum of the evaluation of the uncertainties of the work ( $E_i$ ), the observation and data collection ( $E_d$ ), the computer processing of these points ( $E_p$ ), and those derived from the appreciation of the layout of the work ( $E_r$ ), for which its value ( $E_t = \pm 0.083$  m) and calibration with respect to the three canes of Tortosa represent an error of  $\pm 1.502\%$  with respect to this pattern. The most characteristic values of the data obtained from the apse are statistically analyzed so that the relative error ( $e_r$ ) will be equal to the most probable value ( $c_a$ ) plus or minus the root mean square error of the mean ( $c_{cm}$ ).

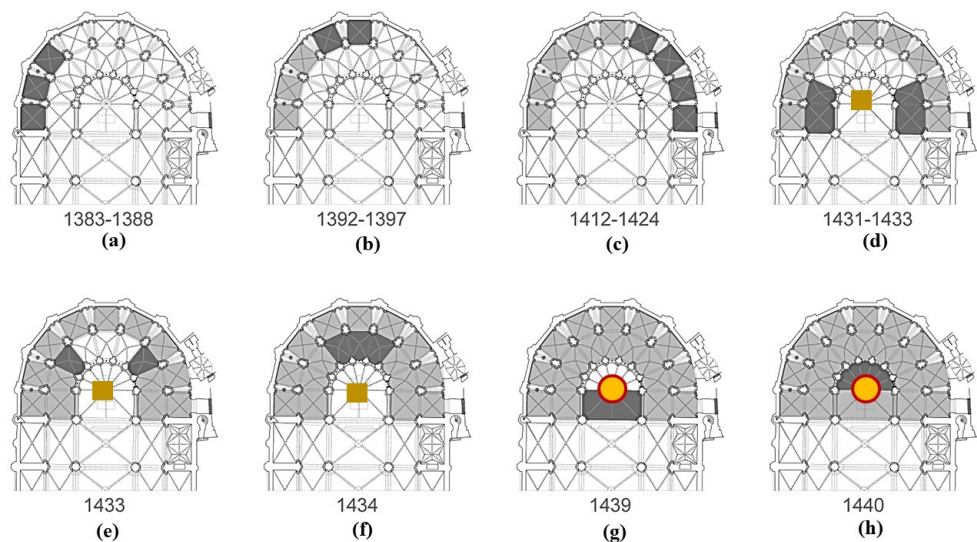
After the statistical analysis of the data carried out by the *magister operis*, possible connections of the sources that inspired this *fabrica* by the Chapter are sought in the existing works of the ACTo. The outline of Guarc (Factory—No. 49) (c. 1345–1380) ( $0.917 \times 0.682$  m) (Figure 8a) is preserved, representing the plan of a project of the apse and part of the body of a cathedral nave, with an annexed chapel with exterior access. On the parchment, a base of auxiliary lines is observed prior to the final outlines (Figure 8b), for which its analysis allows for the establishment of an interpretative methodology of the geometric layout of the apse (Figure 8c) (Lluís i Ginovart et al. 2013, pp. 325–48).



**Figure 8.** (a) Parchment *Mostra d'En Antony Guarç, ACTo Factory—No. 49*; (b) graphic transcription of the parchment's auxiliary lines; (c) metric of the apse layout.

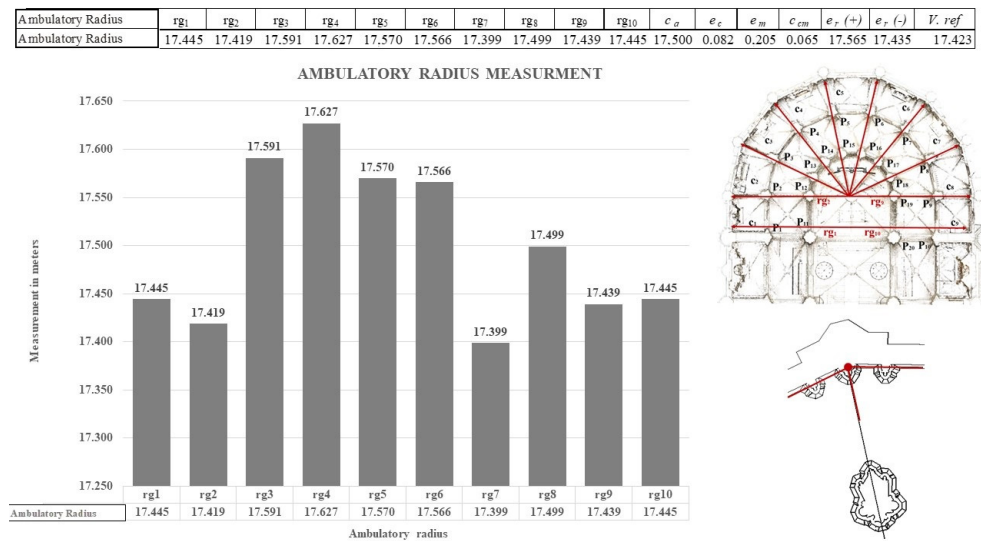
### 3.1. The Metrology of the Floor Plan of the Apse

The apse with a double ambulatory was built between 1374 and 1441, embracing and replacing the previous Romanesque cathedral from the outside. The first phase (1383–1424) involved the construction of the belt of radial chapels, executed in a sequential manner (Figure 9a–c). The second phase determined the construction of the ambulatory (1424–1434) symmetrically covering the axis of the presbytery (Figure 9d–f), which is balanced on the *pilar major* (Figure 9d). Finally, the closure of the presbytery (1435–1441) occurred with the first placement of the keystone on the main pillar (1439) (Figure 9g) and the subsequent construction of the vaults (Figure 9h).



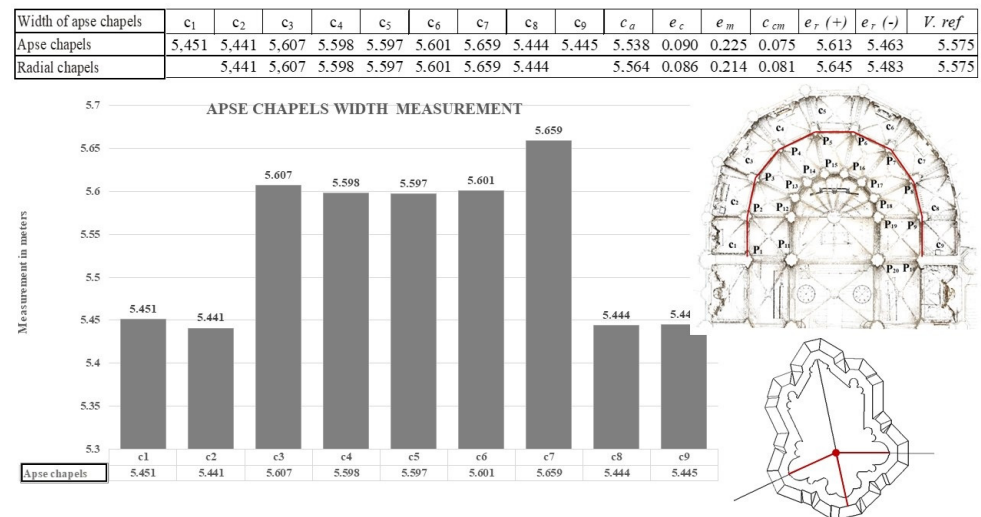
**Figure 9.** An analysis of the constructional evolution of the vaults of the apse of the Tortosa Cathedral (1374–1441); (a–c) (1383–1424); (d–f) (1424–1434); (g,h) (1439–1441).

The total width of the ambulatory is the statistical value of the inner face of the enclosing wall, obtained as the most probable value  $c_a$  ( $rg_{1-10} = 17,500$  m) with a root mean square error of the mean ( $c_{cm} = 0.065$  m), resulting in an interval [17.565–17.435 m] within the reference value (17.423 m), which is equivalent to 75 spans (Figure 10) (Appendix A, Tables A1 and A2).



**Figure 10.** A statistical analysis of the radii of the ambulatory: terrestrial laser scanner (TLS).

To determine the measurement of the nine radial chapels ( $c_1 \dots c_9$ ) and reviewing the auxiliary lines of Guarc’s parchment (T6.2) and (T6.3), the most probable value is ( $c_{a(c1-9)} = 5538$  m). The root mean square error is  $e_c = 0.090$  m, resulting in a relative error ( $e_{r(c1-9)}$ ) with a range of 5.613–5.463 m. Similar results are obtained with the seven radial chapels ( $c_2-c_8$ ), where the reference values yield a range of 5.645–5.483 m, close to 24 spans (5.575 m), which are three *canas* (Figure 11). The centers of the presbytery pillars ( $P_{11}-P_{20}$ ) and the 10 radii ( $rp_1-rp_{10}$ ) are situated at a most probable distance of  $c_a = 6249$  m, with a relative value between the range of 6.278–6.219 m and within the measurement of 6.272 m; they have 27 spans, which is half of the 54 spans of the radius where the radial chapels are traced (Figure 12). The centers of the pillars  $P_{12}$  y  $P_{19}$ , homologous to the new designs of ( $P_{11}-P_{12}$ ) and ( $P_{19}-P_{20}$ ), with displacements  $P_{11}$  and  $P_{12}$ , are 0.107 m and between  $P_{19}$  and  $P_{20}$  (0.168 m).



**Figure 11.** Statistics of the width of the chapels in the apse: terrestrial laser scanner (TLS).

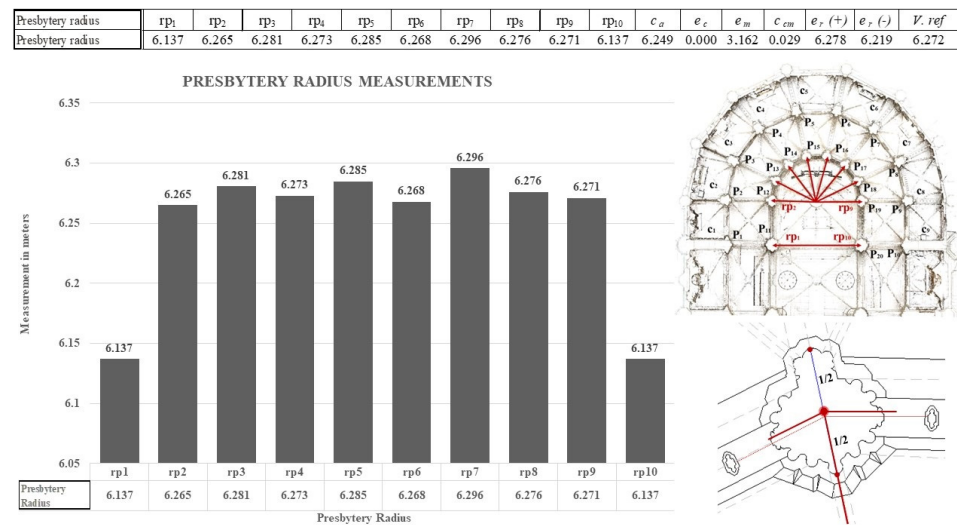


Figure 12. Presbytery radius statistics: terrestrial laser scanner (TLS).

### 3.2. The Metrology of the Section of the Apse

To determine the measurement of the height of the nine radial chapels ( $c_1 \dots c_9$ ), the zenith of the ribbed vault of the radial chapels ( $c_1 \dots c_9$ ) is taken as a reference, and it is located at the neck of the keystone, with the most probable value of the chapels ( $h_{ca(c1-9)} = 10,469$  m) obtaining a relative error ( $e_{r(c1-9)}$ ) in the range of 10.581–10.357 m. The range of the statistical result falls within the metrological measurement of 45 spans (Figure 13) (Appendix A, Tables A1 and A2).

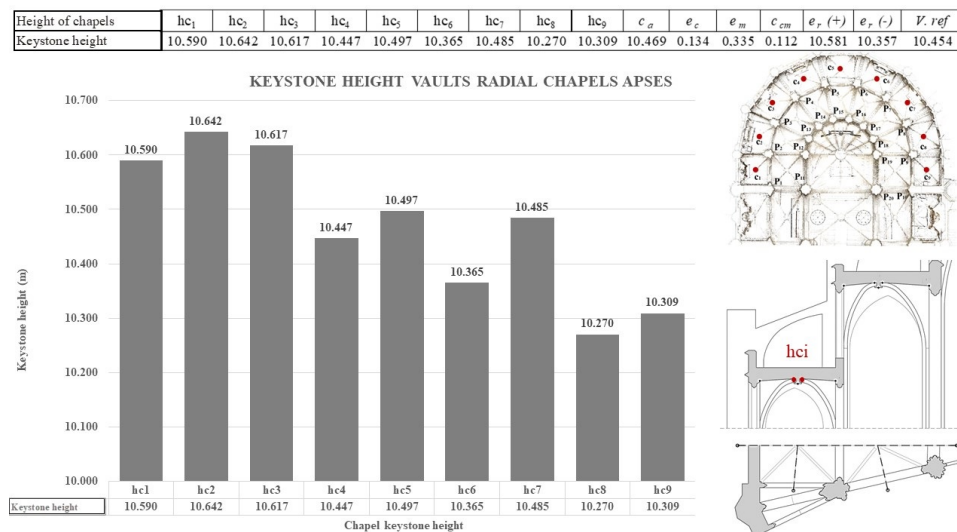


Figure 13. A statistical analysis of the height  $h_{ci}$  of the radial chapels in the apse (TLS).

The height of the nine vaults of the ambulatory ( $h_{d1} \dots h_{d9}$ ) has a most probable value ( $h_{da(c1-9)} = 16.007$  m). The relative error ( $e_{r(c1-9)}$ ) ranges from 16.070 to 15.940 m within the range of 69 spans, and it is close to 70 spans, which is within the methodological error (Figure 14). The section of the presbytery vault does not have a direct measurement since the presbytery keystone casts a shadow over the neck of the constructive element. Taking references from the points (TLS), knowing point A (22.908 m), and given that the arch has a quarter section, we can deduce, by extending said arch, the keystone center, which would be situated at a height of 23.237 m above the presbytery floor. Taking a metrological value of 100 spans (23.230 m), a value similar to the indirect result determined by (TLS) is obtained, and it is where the keystone with the Coronation of the Virgin is located (Figure 15).

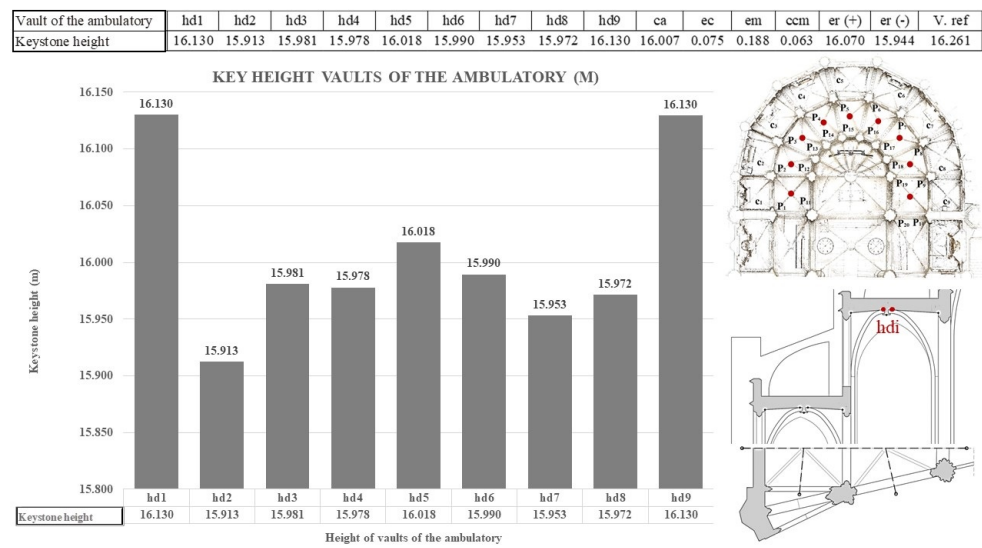


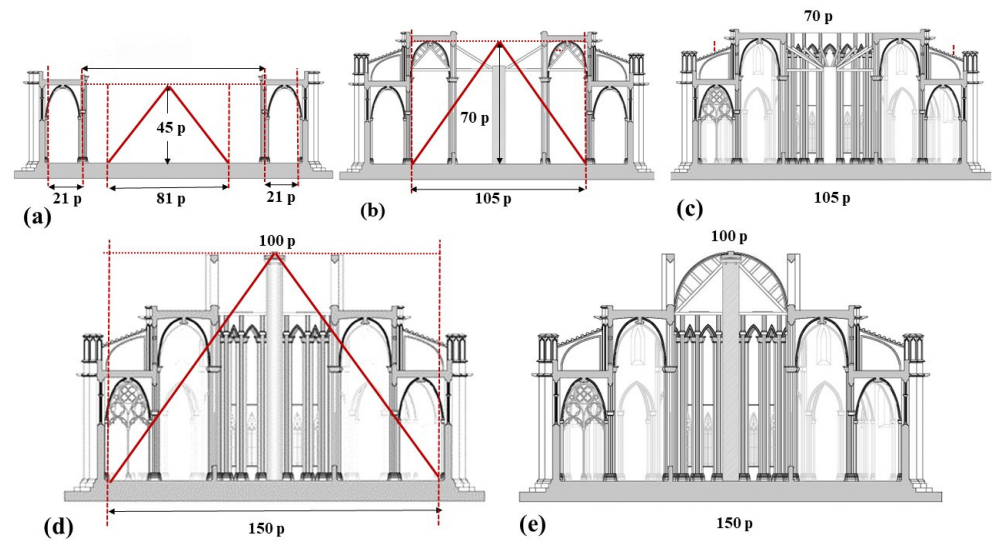
Figure 14. A statistical analysis of the height hdi of the vaults of the ambulatory (TLS).



Figure 15. The main keystone of the apse of the Tortosa Cathedral (1346–1441). Image by the authors.

Analyzing the height of the radial chapels (45 spans) in relation to the proportionality of the Gothic section theory with respect to the metrology of the floor plan, it is observed that at the midpoint of the ambulatory ( $27 + \frac{1}{2} 27 = 40.5$  spans), a proportion is formed between the height of the chapel and the ambulatory ( $45/81$ ), expressing the development of a ratio of  $(9 \div 5)$  (Figure 16a).

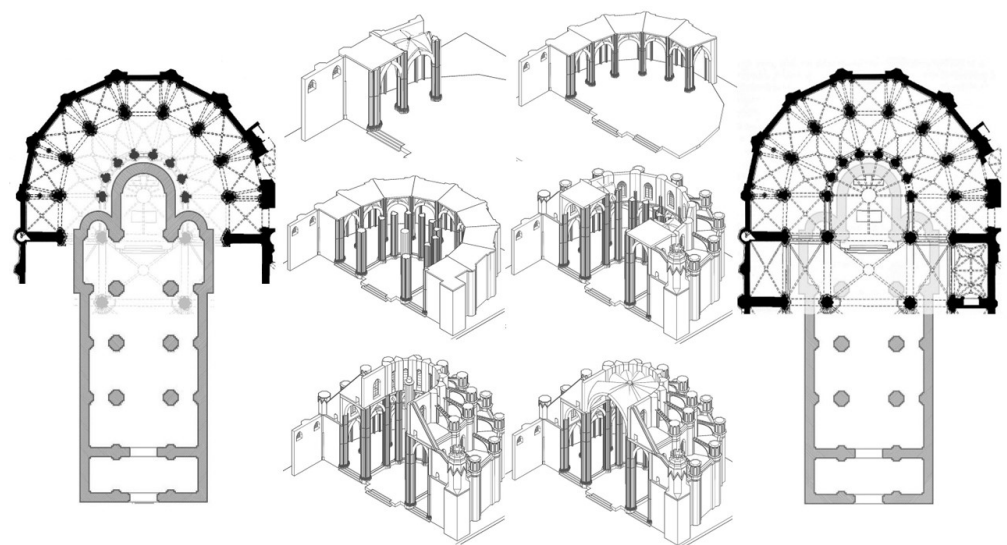
The reference value of the ambulatory keystones [16.070–15.940 m] falls within the range of 69 spans. Considering that the layout of the plan for  $(9 \div 5)$  would need to be  $58 + \frac{1}{3}$  spans, we observe that it is far from the value of the finish of the apsidal chapels. If we check for the ratio  $(9 \div 6)$ , the height of the ambulatory vaults would need to be at 70 spans (Figure 16b,c). The main keystone is situated at 100 spans over a total width of 150 spans from the ambulatory (Figure 16d,e). With these values, we can propose a plan layout of the measurements [81–105–150 spans], elevated in height corresponding to 45–70–100 spans. The presbytery keystone representing the Coronation of the Virgin has a larger crown diameter of  $2.030 \pm 1.502\%$  m, neck diameter of  $1.625 \pm 1.502\%$  m, and a height of  $0.902 \pm 1.502\%$  m, with a total elevation of  $1.280 \pm 1.502\%$  m. The section is  $2.110 \text{ m}^2$  and has a volume of  $3.640 \text{ m}^3$ , with an apparent density of  $26.90 \text{ kp/m}^3$  and a weight of  $97.916 \text{ KN}$  (9.8 Tn).



**Figure 16.** Section displays of the apse of the Tortosa Cathedral (1346–1441). Construction and proportional evolution of the apse; (a) radial chapels; (b,c) ambulatory; (d,e) placement of the keystone of the chancel.

#### 4. A Discussion of the Results

In the design of a Gothic apse, the essential element is the determination of the number of chapels given that, on the one hand, they are the element of uniqueness that has its origin in the Gothic liturgy of the *Prochiron, vulgo Rationale divinatorum officiorum* (1291), written by the Bishop of Mende, Gulielmus Durandus (1230–1296). This liturgy is known in Tortosa through ACTo 58 from the end of the 13th century and the incunabula of Rome (1477) (ACTo No. 258) and Venice (1482) (ACTo No. 290). On the other hand, the chapels are a modular element in construction as they allow for economic sponsorship and act as a buttress element (Puig i Cadafalch 1923, pp. 65–87). From a metric point of view, the radial and lateral chapels must be equal since they are part of the sequence of the pilgrimage that circumnavigates the apse (Font i Carreras 1891, pp. 9–14). In the case of Tortosa, as with the Augustinian canonical church of Girona, in 1312, nine chapels were built in its apse (Street 1865, pp. 318–39)<sup>16</sup>, which was obligated by a contract, and seven of them are located within the perimeter of the circular crown of the presbytery; the other two are located on the axis of the existing Romanesque cathedral (Figure 17).



**Figure 17.** The constructional evolution of the apse of the Tortosa Cathedral (1346–1441).

#### 4.1. The Metrology of the Apse and Augustine of Hippo

The magister operis had to solve three geometric problems: first, the utilization of a method for drawing the heptagon; second, constructing the geometric figure without knowing the center because it was occupied by the ecclesia vetulam; and finally, proportionally solving the relationship between the radial chapels and the two chapels in the straight section. In the bijective analysis, with respect to *theorica y practica*, which is the comparative reading between the construction and the ACTo, we have direct sources in the inventory of *De civitate Dei* [5], ACTo 20, *Supra Psalterium* [6]. We also have *Enarrationes in Psalmos* (Baiges 1999, pp. 3–20) and the *De Trinitate* (ACTo 55, fol. 1r–172r); the *De libero arbitrio* (ACTo 86, fol. 33v–59v); and the *De genesi ad litteram* (ACTo 86, fol. 59v–124v) (Denifle and Chatelain 1896, pp. 7–26).

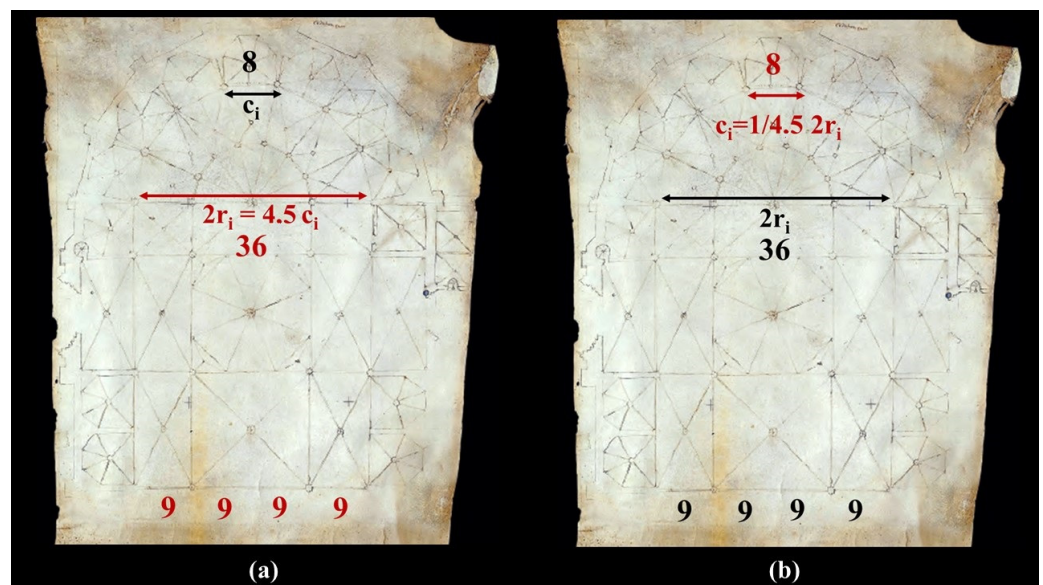
The number seven does not appear in the early works of the *Soliloquia* (386) and *De ordine* (386). It is in the *De quantitate animae* (388) that he introduces it more and more into his discourse due to the influence that came to him through the catechesis for baptism and the sermons of St. Ambrose (340–397) (Cilleruelo 1953, pp. 510–11). In *De Civitate Dei* (ACTo 20), he refers to the number seven in relation to the seventh day as fullness and rest after Creation (*Civ Dei*. XI.31) (ACTo fol 168r–169v)<sup>17</sup>. On the seventh day, the same is seven times, and it manifests as God’s rest and the sanctification of this day. The author warns that the first total odd number is three, which, together with four, forms the septenary.

In *De genesi ad litteram* (ACTo 86, fol. 59v–124v), the reference to the number seven is made through the number three, which is part of this number, along with eight and nine. Seven can be divided into three and four (*De Gen. ad Litt.* IV.2,2)<sup>18</sup>. In *Enarrationes in Psalmos* (392), it is related to the Ten Commandments, which are divided between the three that show love to God and the seven that show love to their neighbor (*Enr. Psal.* 32. 2, 6)<sup>19</sup>.

Regarding the seven radial chapels, the figure of the heptagon, as recognized by Bovelles, does not appear in Euclid’s “Elements”. The *Elementa* as translated by Adelardo of Bath (1075–1166) in 1142 considers the layout of regular polygons in Book IV<sup>20</sup>, but it fails to mention either the heptagon or the mathematical syntax in Ptolemy’s (ca. 85–165) *Almagesto*, which was translated by Gerardo of Cremona (1114–1187) around 1175<sup>21</sup>. The heptagonal layout by means of geometrical instrumentation was first refuted by Kepler (1571–1630) in his *Harmonices Mundi, Libri V* (1619), in the Propositio. XLV (Kepler 1619, 32–40)<sup>22</sup>, and later by Gauss (1777–1855) at the end of his *Disquisitiones Arithmeticae* in Section VII, Propositions 361–366 (Gauss 1801, pp. 454–63).

We know the methodology of constructing the heptagon through the *Geometria Deutsch* (1472), attributed to Hans Hösche von Gmünd (fl. 1472) (Heideloff 1844, pp. 96–97), and the *Geometria Deutsch* (1488) by Matthäus Roriczer (+c. 1495) (Roritzer 1999, pp. 56–60). In these works, the side of the heptagon is determined as the height of an equilateral triangle with a side equal to the radius of the circumference. This method has come down to us through *Underweysung der Messung* (1525) by Albrecht Dürer (1471–1528), a consequence of the corollary of the construction of the pentagon (LII.15) with a value of  $\sqrt{3}/2$  (Dürer 1525, 26r–27v). The origin of this method can be traced back to the *Kitāb fī mā yahtāju al-ṣāni’ min al-a’ mā al-handasiyya* (c. 993–1008) (*Book on those geometric constructions which are necessary for craftsmen*) by Mohammad Abu’l-Wafa Al-Buzjani (940–998), and its reception in Latin Europe occurred through Ibn Yunus, Kamal al-Din (1156–1242) in the court of Emperor Frederick II (1194–1250) (Raynaud 2012, pp. 34–83). Guarc’s method determines the diameter of the circumference through the side of the regular polygon and its ratio (18:8), unlike Euclid’s “Elements,” Book IV, where the side is a consequence of the inscribed circumference (Heath 1908, pp. 88–111). Knowing the measurement of the chapel ( $c_i$ ) results in the determination of the ambulatory ( $4,5 c_i$ ) (Figure 18a), or vice versa is also possible given the width ( $2 r_i$ ) and the radial chapel ( $1/4,5 2r_i$ ) (Figure 18b).

We can speculate about the geometric method that might have been used to replant the radial chapels based on Abu'l-Wafa's (c. 993–1008) approach, starting from the height of the triangle ( $\sqrt{3}/2 = 0.866$ ), or other medieval approximations such as the 7:6 ratio provided by Gerbert of Aurillac (c. 950–1003) with a measurement of 0.8571 (Bubnov 1899, pp. 43–45). Gabriele Stornaloco in 1391 utilized [8:7] 0.875 (Ackerman 1949, pp. 84–111) or the 9:8 ratio of 0.889 provided by Antoni Guarc (c. 1345–1380). The relationship between the side of the tetradecagon ( $c_i$ ) and the radial axes of the ambulatory ( $r_{di}$ ) is analyzed. The most probable values for the radial chapels ( $c_{a(c2-8)} = 5564$  m) and the radius ( $c_{a(rd1-9)} = 12,553$ ) have a relationship  $(c_{a(c2-8)})/0.5 c_{a(rd1-9)} = 0.886$ , which is closer to Guarc's resolution. The chapel has a statistical value of three canes and is situated over a radius of 54 spans. Guarc's ratio is mentioned in Macrobio's *Comentarii In Somnium Scipionis* in ACTo 236 (fol. 1r. 61v) with the term "epogdo" (9/8) (ACTo 236 fol. 36v), as well as in the translation and commentary of Plato's "Timaeus" by Calcidio (fl.350) where the epogdo is expressed as  $(1 + 1/8)$  (ACTo 80, fol. 150r–150v). This completes the issue with a fragment from Book VII of Marcius Capella's "Geometria" (fl. 430) (ACTo 80, fol.160v–161r) where the commensurable measure is *rhētós*.



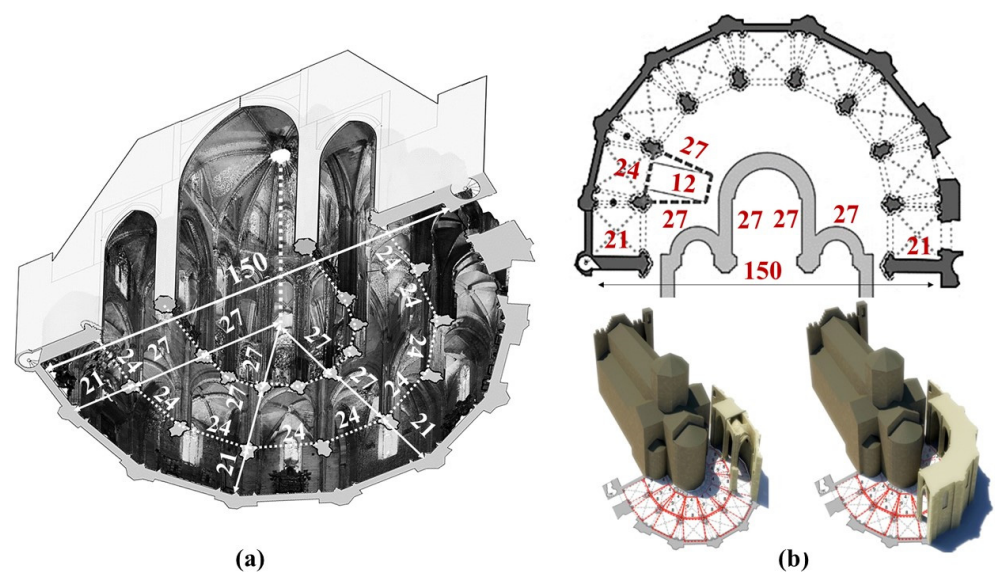
**Figure 18.** The arithmetic–geometric development of Antonio Guarc (ca. 1345–1380). (a) Knowing the average of the chapel, the width of the ambulatory is deduced; (b) Knowing the width of the ambulatory, the width of the radial chapel is known.

The centers of the pillars in the presbyteries ( $P_{11}$ – $P_{12}$ ) and ( $P_{19}$ – $P_{20}$ ) of the new design correspond to a radius of 27 spans, half of the 54 spans of the presbytery ( $2 \times 27$ ). In turn, the layout of the ambulatory was carried out at 108 spans ( $2 \times 2 \times 27$ ), a measurement related to the number nine in Book XVIII (ACTo 20, fol. 294v–295r). This reference takes into account the number of the twenty-seven verses, defining this number as three raised to the cube since three times three is nine and in turn transforming it from width to height: three times nine is twenty-seven (*Civ. Dei* XVIII.23,2) (ACTo 20, fol. 295r)<sup>23</sup>. Nine is also referenced in "De Genesi ad Litteram" (ACTo 86, fol. 59v–124v) in relation to its divisors of three and six, recognizing the three parts of three, which is one-third of nine (*De Gen. ad Litt.* IV.2, 2)<sup>24</sup>.

The width of the cathedral of 150 spans is determined using the statistical value of its radii ( $17.423 \pm 0.065$  m (17.423 m)), which is equivalent to 75 spans. At the end of the codex *Supra psalterium* [6] from a reference of the old inventory, which we have interpreted as *Enarrationes in Psalmos* in the commentary on Psalm 150, Augustine of Hippo provides a substantial apology for this number<sup>25</sup>. Firstly, he considers the number 15, from which 150 is formed, since the number 15 in relation to the simple numbers is a relation of the number 150 with respect to the tens because it comprises fifteen times

ten. The number 15 symbolizes the conformity of the two Testaments. In the Old Testament, the Sabbath is observed, which signifies rest; in the New Testament, the Lord's Day signifies resurrection. The Sabbath is the seventh day of the week and is Sunday, which immediately follows the seventh. The number 50 contains a great sacrament within itself, as it consists of weeks with the addition of one day, as if it were the 8th, which completes the number 50. Seven times seven results in the number forty-nine, to which one is added to form fifty. The number 50 contains great symbolism: Counting from the day of the Lord's resurrection, on the fiftieth day, the Holy Spirit came upon those who were gathered in Christ (*Enr. Psal.* 150, 1). He adds that 50 refers to penance, 100 to mercy and judgment, and 150 to the praise of God in His saints as we progress towards eternal life (*Enr. Psal.* 150, 3). A similar conception appears in the *Liber numerorum qui in sanctis Scripturis occurrunt* (612–615) (*Book of Numbers*) attributed to Isidore of Seville (c. 560–636), where this number takes on significance, through the Holy Scriptures, as a perfect number and of those who are predestined by God toward eternal life. (*Lib. Num.* 27,109)<sup>26</sup> (Pardillos 2000, pp. 285–304).

The metrological condition is based on the chapel with 24 spans ( $3 \times 8$  spans) and 3 *canas*, with a total width of 150 spans [21–27–27–27–21] (Figure 19a). Given that the radial chapels have a depth of 21 spans, the metrological construction is  $3 \times 7$ ,  $3 \times 9$ ,  $3 \times 9$ ,  $3 \times 9$ ,  $3 \times 9$ ,  $3 \times 7$  spans, allowing for the tracing of the apse without the circumference that inscribes it (Figure 19b). In the background, there is an intrinsic relationship between seven and nine, which is recognized in *De Musica libri sex* (387), in the numerical articulation of the half-feet:  $(2 + 2) + (2 + 1) = 7$  and  $(2 + 2) + (2 + 3) = 9$  (*De Mus.* V. 4, 7)<sup>27</sup>.

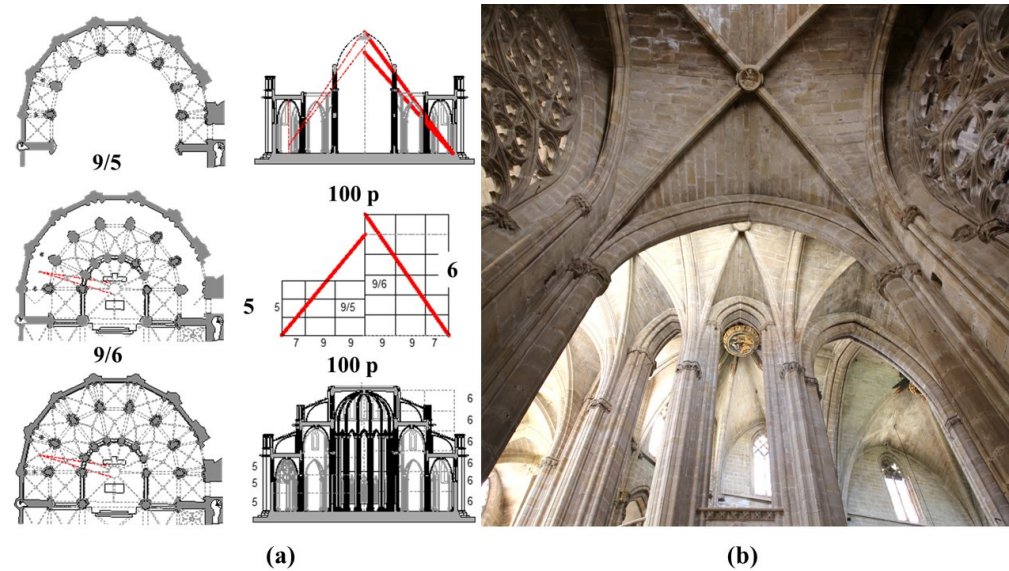


**Figure 19.** The layout of the apse of the Tortosa Cathedral (1346–1441). (a) Metrological construction of the apse of 150 spans with a radial chapel of 24 spans; (b) Polygonal layout of the ambulatory without the need to determine the center of the apse.

#### 4.2. The Metrology of the Apse Section and Augustine of Hippo

Analyzing the height of the radial chapels (45 spans) in relation to the proportionality of the theory of the Gothic section with respect to the metrology of the floor, an initial relationship of  $(9 \div 5)$  is expressed. The reference value of the keys of the ambulatory [16.070–15.94 m] is within the range of close to 70 spans, with a floor layout of  $(9 \div 6)$ . Finally, the main key is situated at 100 spans above a total width of 150 spans from the ambulatory relative to the base of the inner wall of the facade. With these values, we can propose a floor layout of the measurements [81–105–150 spans], elevated in height in a corresponding metrological manner to [45–70–100 spans] (Figure 20a). The evidence of a change in proportionality of the model by Pascacio Xulbi (b. 1383–1441) and Juan Xulbi (b. 1416–1428) with respect to the initial model entailed a structural and visual change,

where the image of the Coronation of the Virgin will not only have a central vision but will also visually preside over the chapels of the ambulatory in a centripetal manner (Figure 20b). At the change in section and the proportion 3:2, *sesquialtera* appears in the commentary of *Timaeus* by Chalcidius (fl.350) (ACTo 80 146r–155 v), as in *Comentarii In Somnium Scipionis* of ACTo 236 (fol. 1r. 61v) by Macrobius.



**Figure 20.** (a) The evolution of the apse section of the Tortosa Cathedral; (b) presbytery key from the radial chapels.

The *De civitate Dei* (413–426) addresses the judgment in Book XX (ACTo 20 fol. 333r–359r), dedicated to the two resurrections and the thousand years of the apocalypse (*Civ. Dei* XX.7) (fol. 337v–338v). It uses the thousand years to designate the duration of the world, employing it as the fullness of time. The number one thousand is the cube of ten, and ten times ten is one hundred. This is a flat figure, and to make it solid, it is necessary to multiply one hundred by ten, and we already have one thousand (XX.7, 2)<sup>28</sup>. The carving and roughing of the key attributed to Bartomeu Santalínia (b. 1420–1440) exhibit a neck ( $1.625 \text{ m} \pm 1.502\%$ ) within the range of 7 spans and a height ( $1.280 \text{ m} \pm 1.502\%$ ) within 5.5 spans, where the ten vaults and the eleven transverse arches converge. The crown of the ten angels surrounding it has a measurement ( $203.00 \text{ m} \pm 1.502\%$ ). Given the geometric difficulty of stone carvings, hypothetically, it could be thought to have had a diameter of ten spans such that the measurement of key 10, when multiplied by 10, is situated at 100 spans, and when it is multiplied by 10, it becomes 1000, representing the fullness of time.

The iconography of the Coronation of the presbytery corresponds to the enthroned Virgin, situated to the right of her Son (Puigarnau 2023, p. 914), who is being crowned with glory and honor and holds an orb in her hand (Hebrews, 2:9); she also exhibits a gesture of reverence before the King of kings and Lord of lords (Revelation, 19:16), both sharing the throne surrounded by ten angels held in their flight (Figure 21b).



**Figure 21.** (a) *Horae ad usum parisiensem* (1375–1425)<sup>29</sup>; (b) keystone of the Tortosa Cathedral; (c) *De civitate Dei* book XXII (s. XV)<sup>30</sup>.

The Coronation of the Virgin concludes with the episodes of her Dormition and her Assumption into heaven, which are frequent themes in iconography attributed to Melito of Sardis (+c. 180)<sup>31</sup>. It was popularized in the passage *Summa aurea* (1261–1266) by Jacobus de Voragine (ca. 1229–1298), *Assumptio Beatae Mariae Virginis* Cap. CXIX, which states, “Come from Lebanon, you who are to be crowned.” (Voragine 1999, pp. 479–83), and this was referred to in the 1420 cathedral’s inventory: *Item, liber vocatus Flos sanctorum, in pergamento, in latino, qui fuit honorabilis domini decani Dominici Meseguer* [38]; *Item, Summa Aurea, in pergamento, cum littera minuta, spessa, cum cohoptis virnilis* [40]; and *Item, quidam liber vocatus Flos sanctorum* [48] (Baiges 1999, pp. 3–20). The image of the cathedral (Figure 21b) is consistent with the illumination of the *Horae ad usum parisiensem* (1375–1425) (BnP, NAL 3093-2, fol. 75v) (Figure 21a).

The Coronation also appears to illustrate Book XXII of *De Civitate Dei*, for which its object is the eternal happiness of the saints in order to attain the City of God, thus giving solidity to the faith in the resurrection of bodies, where it is also linked to the Final Judgment, given that the crown is of life (Revelation, 2:10) and where the purely human woman is the first to fulfill God’s project for the divinization of man.

The ten angels surrounding the coronation appear in *De Civitate Dei* (BnF, Franc. 24 fol. 262v) (15th century) (Figure 21c), accompanied by bishops, the Pope, and the Saints. The keystone of the presbytery is protected by eleven winged dragons arranged on the crossing arches converging at the main keystone. The meaning of the dragon, symbolizing the combat to be overcome by Virtue, is that it is a protective element of the temple and the faithful who seek refuge in it (Figure 22).

The placement of the keystone took place with a Solemn Mass celebrated on 27 September 1439. The chosen date is the first Sunday after the forty days of the Assumption, which could have been better chosen by taking the *Legenda aurea* as a reference: “When my dormition occurred, all the apostles were by my side. They were precisely the ones who, with utmost reverence, carried my body to the tomb. Forty days after my death, I resurrected” (Voragine 1999, p. 482). The document says, “Today, Sunday, 27 September 1439, with the Altarpiece of the Cathedral open and well adorned, all the bells ringing, which had rung last night for the occasion... Here the office was performed, and the Mass very solemnly of the Assumption of the Virgin Mary” (O’Callaghan 1887, pp. 17–20).



**Figure 22.** The Coronation of the Virgin surrounded by eleven guardian dragons.

## 5. Conclusions

The Gothic cathedral of Tortosa, conceived from 1346 onwards, is the last of the Catalan Gothic cathedrals. Through the results obtained, its peculiar characteristics are visualized both in terms of its architectural order and its construction systems within the southern Gothic context. The principles in ecclesiastical replacement since 1155 by Bishop Godofredo (+1165) of *vivere sub regula Beati Agustini, et iuxta consuetudines Ecclesiae Sancti Ruffi*, render the figure of St. Augustine present both in intellect and in the iconography of the vault keystones. The abundant work still preserved in the eleven codices and thirty-two works is the greatest idolatry of the promoters of the cathedral. With the principle of numbers used as the ordering of creation by St. Augustine on the one hand and, on the other hand, the results obtained via quantitative methodology focused on the constructive reality analysis of the Tortosa Cathedral through close-range photogrammetry (CRP) and terrestrial laser scanning (TLS), it can be concluded that the construction was executed with great precision and with an error of less than 1.502%.

These direct sources coincide with the large dimensions of the cathedral: 150 span wide, 100 span high, and keystones measuring 10 spans in diameter, surrounded by 10 angels and the 7 radial chapels. The main works of Chalcidius and Capella (ACTo No. 80) and Macrobius (ACTo 236) also appear in this library. What is particularly interesting is the tonal ratio (9/8) of the epogdo used by Macrobius' *Comentarii In Somnium Scipionis*, who was the teacher of Augustine of Hippo and who appears in the codex (ACTo 236 fol. 36 v), and the ratio is used for the layout of the seven radial chapels. This thesis is in line with the principles of classical art historiography originating from Wilhelm Worringer (1911), Erwin Panofsky (1951), and especially Otto von Simson in Order (1956).

**Author Contributions:** C.L.-T.: Formal analysis, Research. J.L.i.G.: Conceptualization, Methodology. I.U.-B.: Draft manuscript review. R.G.-V.: Draft manuscript writing, Manuscript review and editing. All authors have read and agreed to the published version of the manuscript.

**Funding:** School Architecture Campus Barcelona, Universitat Internacional de Catalunya, Barcelona, Spain.

**Institutional Review Board Statement:** Not applicable.

**Informed Consent Statement:** Not applicable.

**Data Availability Statement:** Special thanks to the Arxiu Capítular Tortosa ACTo.

**Acknowledgments:** We thank Canon Archivist Josep Alanya i Roig for his assistance in accessing documentation during many years of work at the Cathedral of Tortosa; Agustí Costa i Jover and Sergio Coll i Pla from the Rovira i Virgili University (URV); and Josep M<sup>o</sup> Puche i Fontanilles from the Catalan Institute of Classical Archaeology (ICAC), who collaborated in some of the surveys carried out at the cathedral in the last decade.

**Conflicts of Interest:** The authors declare no conflicts of interest.

**Abbreviations**

ACTo	Archivo Capitular Tortosa
AHCTE	Arxiu Històric Comarcal Terre de l’Ebre
BAC.	Biblioteca Autores Cristianos Obras de San Agustín
BNE	Biblioteca Nacional España
BnP	Biblioteca Nacional Paris
(Civ. Dei.)	De civitate Dei (413–426)
(Conf.)	Confessionum, libri tredecim (397)
CRP	Close-range photogrammetry
(De Gen. ad litt.)	De Genesi ad Litteram libri duodecim (401)
(De Mus.)	Musica libri sex (387)
(De Ord.)	De ordine, libri duo (386)
(Enr. Psal.)	Enarrationes in Psalmos (392)
(Lib. Num.)	Liber numerorum qui in sanctis Scripturis occurrunt (612–615)
PL	Patrologia Latina. Jacques Paul Migne. Patrologiae Cursus Completus. Series Latina
TLS	Terrestrial laser scanner

**Appendix A**

**Table A1.** The measurements of the magnitudes of the pillars of the apse.

APSE PILLARS DIMENSIONS										
Dimensions of Radial Chapel Pillars										
Pillars	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
F ex/m,	1.049	1.923	1.992	1.980	2.004	2.002	2.010	2.002	1.896	1.046
F ey1/m,	2.387	0797	0.861	0.767	0.765	0.748	0.788	0.738	0.806	2.362
F ey2/m,		0.903	0.780	0.769	0.783	0.732	0.760	0.763	0.795	
F s, /m <sup>2</sup>	1.611	1.677	1.755	1.742	1.769	1.684	1.787	1.709	1.681	1.593
B ex/m,	1.375	2.456	2.551	2.540	2.564	2.562	2.570	2.562	2.456	1.375
B ey1/m,	1.464	1.079	1.029	1.044	1.048	1.012	1.040	1.043	1.075	1.458
B ey2/m,	1.483	1.086	1.063	1.052	1.060	1.027	1.069	1.018	1.087	1.461
B s, /m <sup>2</sup>	3.013	3.695	3.726	3.741	3.782	3.668	3.807	3.689	3.695	2.984

Table A1. Cont.

APSE PILLARS DIMENSIONS										
Dimensions of presbytery chapel pillars										
Pillars	P11	P12	P13	P14	P15	P16	P17	P18	P19	P20
F ex/m	1.928	1.641	1.663	1.653	1.624	1.682	1.634	1.666	1.652	1.928
F ey1/m	0.964	0.966	0.756	0.75	0.774	0.77	0.776	0.744	0.758	0.947
F ey2/m	0.954	0.699	0.774	0.77	0.777	0.735	0.787	0.774	0.987	0.963
F s, /m <sup>2</sup>	2.364	1.563	1.41	1.391	1.391	1.405	1.41	1.402	1.642	2.354
B ex/m	2.238	2.201	2.23	2.213	2.034	2.092	2.194	1.911	1.902	2.238
B ey1/m,	1.119								0.833	
B ey2/m,	0.954							0.849		1.118
B s, /m <sup>2</sup>										

Table A2. The measurement of the layout in the plan of the apse.

MEASUREMENTS APSE OF THE CATHEDRAL OF TORTOSA																
Chapels of the Ambulatory																
Width of apse chapels	c1	c2	c3	c4	c5	c6	c7	c8	c9	ca	ec	em	ccm	er (+)	er (-)	V. ref
Apse chapels	5.451	5.441	5.607	5.598	5.597	5.601	5.659	5.444	5.445	5.538	0.09	0.225	0.075	5.6132	5.463	5.575
Radial chapels		5.441	5.607	5.598	5.597	5.601	5.659	5.444		5.564	0.086	0.214	0.081	5.6448	5.483	5.575

Table A2. Cont.

MEASUREMENTS APSE OF THE CATHEDRAL OF TORTOSA																	
Apse ambulatory radius																	
Apse ambulatory radius	rg <sub>1</sub>	rg <sub>2</sub>	rg <sub>3</sub>	rg <sub>4</sub>	rg <sub>5</sub>	rg <sub>6</sub>	rg <sub>7</sub>	rg <sub>8</sub>	rg <sub>9</sub>	rg <sub>10</sub>	c <sub>a</sub>	e <sub>c</sub>	e <sub>m</sub>	c <sub>cm</sub>	e <sub>r</sub> (+)	e <sub>r</sub> (-)	V. ref
Gyrola radius	17.445	17.419	17.591	17.627	17.570	17.566	17.399	17.499	17.439	17.445	17.500	0.082	0.205	0.065	17.565	17.35	17.423

Presbytery Radius																	
Presbytery radius	rp <sub>1</sub>	rp <sub>2</sub>	rp <sub>3</sub>	rp <sub>4</sub>	rp <sub>5</sub>	rp <sub>6</sub>	rp <sub>7</sub>	rp <sub>8</sub>	rp <sub>9</sub>	rp <sub>10</sub>	c <sub>a</sub>	e <sub>c</sub>	e <sub>m</sub>	c <sub>cm</sub>	e <sub>r</sub> (+)	e <sub>r</sub> (-)	V. ref
Presbytery radius	6.137	6.265	6.281	6.273	6.285	6.268	6.296	6.276	6.271	6,37	6.249	0.000	3.162	0.029	6.278	6.219	6.272

**Table A3.** The measurements of the section of the apse vaults.

APSE VAULT DIMENSIONS								
Radial Chapel Vaults								
Space	SECTION SL (Longitudinal)				SECTION ST (Transversal)			
Chapel	c	hci	a	e	c	hci	a	e
c1	10.192	10.590	10.358	10.231	10.192	10.590	10.277	10.240
c2	10.253	10.642	10.367	10.270	10.253	10.642	10.264	10.298
c3	10.245	10.617	10.378	10.163	10.245	10.617	10.336	10.283
c4	10.101	10.447	10.533	10.188	10.101	10.447	10.270	10.313
c5	10.093	10.497	10.475	10.235	10.093	10.497	10.364	10.359
c6	9.945	10.365	10.441	10.224	9.945	10.365	10.274	10.312
c7	10.084	10.485	10.409	10.314	10.084	10.485	10.270	10.314
c8	9.851	10.270	10.479	10.287	9.851	10.270	10.327	10.376
c9	9.900	10.309	10.475	10.178	9.900	10.309	10.443	10.303

The diagram illustrates the structural layout of the apse vaults. It includes two longitudinal sections (SL) and two transversal sections (ST). The longitudinal sections show the profile of the vaults with measurement points labeled 'at', 'c', 'et', 'bt', 'hci', and 'dt'. The transversal sections show the plan view of the vaults with measurement points labeled 'at', 'bt', 'c', 'dt', 'et', and 'St'. The vaults are shown in a perspective view, highlighting their radial arrangement around the apse.

Vaulting vaults								
Space	SECTION SL (Longitudinal)				SECTION ST (Transversal)			
Gyrola	c	hdi	a	e	c	hdi	a	e
d1	16.30	16.650	16.240	16.257	16.130	16.650	16.275	16.313
d2	15.913	16.439	16.252	16.323	15.913	16.439	16.289	16.310
d3	15.981	16.493	16.253	16.334	15.981	16.493	16.313	16.322
d4	15.978	16.498	16.291	16.334	15.978	16.498	16.336	16.374
d5	16.018	16.533	16.286	16.367	16.018	16.533	16.383	16.405
d6	15.990	16.500	16.306	16.342	15.990	16.500	16.440	16.342
d7	15.953	16.461	16.264	16.369	15.953	16.461	16.328	16.318
d8	15.972	16.479	16.290	16.342	15.972	16.479	16.317	16.312
d9	16.130	16.652	16.326	16.319	16.130	16.652	16.307	16.315

**Notes**

- <sup>1</sup> Indirect methods allow for the measurement of space; thus, it is possible to observe and measure points of the factory that are not accessible with high precision.
- <sup>2</sup> (Augustine of Hippo 1845, PL32 col. 1015; 1969, BAC3 678–679).
- <sup>3</sup> (Augustine of Hippo 1845, PL32 col. 968–969; 1969, BAC3 617).

- 4 (Augustine of Hippo 1845, PL32 col. 705; 1958a, BAC2 185).
- 5 (Augustine of Hippo 1845, PL32, col. 1014; 1969, BAC1 676).
- 6 (Augustine of Hippo 1845, PL32, col. 1001; 1969, BAC1 647–648).
- 7 (Augustine of Hippo 1865a, PL34, col. 299–300; 1957, BAC15 731).
- 8 (Augustine of Hippo 1845, PL32, col. 1014; 1969, BAC1 678–679).
- 9 (Agustín de Hipona 1845, PL32, col. 1254; Augustine of Hippo 1962, BAC3 285).
- 10 The concept of love in St. Augustine, from its double dimension known as the love of use (uti) and the love of enjoyment (frui), (López 2016, pp. 104–25.)
- 11 Bouvelles, Tractatus varii de rebus philosophicis, conserved in Tortosa ACTo. 300. The compilation has two parts; the first part, of a scientific philosophical nature, is composed of Liber de intellectu, Liber de sensu, Liber de nichilio, Ars oppositorum, Liber de generatione, Liber de Sapient, Liber de duodecim numeris, and Epistole complures. The second part is dedicated to mathematics, numbers, and polygonal figures: Insup mathematicuopus quadripartitu, De Numenis perfectis, De Mathematicis Rosis, De Geometricis Corporibus, and De Geometricis Supplementis.
- 12 BnF. Bibliothèque Nationale de Paris, Bibliothèque nationale de France. Département des Manuscrits, Français 19093, <https://archivesetmanuscrits.bnf.fr/ark:/12148/cc482952> (accessed on 11 January 2024).
- 13 <http://classes.bnf.fr/villard/feuille/index.htm> (accessed on 11 January 2024).
- 14 See note 13 above.
- 15 Felipe II (1527–1598) Cortes de Monzón (1585). Through Chapter 89 of the Courts of Monzón (1585) King Philip II (1527–1598) unified the metric criteria. The prosecutors of Tortosa, under oath on 24 July 1593, refer to the documentation about the reduction in the cana of Tortosa to Barcelona. This is a noted example.
- 16 Capitulum Gerundense more solito congregatum statuit, voluit et ordinvit, quod caput ipsius ecclesiae de novo construeretur ethadificaretur, et circum circa ipsum caput, novem capellae fierunt et in dormitorio veteri, fieri sacristia. This is a noted example.
- 17 (Augustine of Hippo 1864, PL41 col. 644–645; 1958b, BAC16 769–771).
- 18 (Augustine of Hippo 1865a, PL34, col. 206–207; 1957, BAC15 723).
- 19 (Augustine of Hippo 1865b, PL36 col. 281–282; 1964, BAC19 432).
- 20 Euclid, Elements, Book IV; the triangle (IV.5), the square (IV.6 to 9), the pentagon (IV.11 to 14), the hexagon (IV.15), and the fifteen-sided polygon (IV.16) in the first edition, (Heath 1908, pp. 88–111).
- 21 Figure of the square, pentagon, hexagon, decagon, and dodecagon: see book I, (Toomer 1984, pp. 35–74).
- 22 This is a noted example. Johannes Kepler raised the problem of the constructability of the heptagon with a straightedge and compass in the fifth book of his *Harmonices mundo* (1619). There, he argued that the figure is not an entity capable of being known, since its formal description is impossible. It therefore can be constructed neither by the human mind nor in the infinitude of God, the Creator. Kepler’s work had an inquisitorial response from the Dominican friar Ignacio Munoz Pinciano (ca. 1608–1685), who published the *Manifiesto Geométrico* (1684), (Lluís i Ginovart and Lluís-Teruel 2023, pp. 108–17).
- 23 (Augustine of Hippo 1864, PL41 col. 580–581; 1958b, BAC16 1282–1283).
- 24 See note 18 above.
- 25 (Augustine of Hippo 1865c, PL37 col. 1960–1964; 1967, BAC22 920–927).
- 26 (Isidoro de Sevilla 1862, PL82 col. 200).
- 27 (Augustine of Hippo 1845, PL32, col. 1149–1150; 1988, BAC32 254–256).
- 28 (Augustine of Hippo 1864, PL41 col. 667–668; 1958c, BAC16 1457).
- 29 <https://portail.biblissima.fr/fr/ark:/43093/mdata340381b339947a844b4603a423bed7f8994228d8> (accessed on 11 January 2024).
- 30 <https://portail.biblissima.fr/fr/ark:/43093/ifdata6444c40b2504672da19c58928e874dd777e9b78c> (accessed on 11 January 2024).
- 31 (Pola 2003, p. 48).

## References

- Ackerman, James S. 1949. “Ars Sine Scientia Nihil Est”. Gothic Theory of Architecture at the Cathedral of Milan. *The Art Bulletin* 31: 84–111.
- Alonso, Manuel. 1955. *De Scientiis. Compilación a base principalmente de la de Al-Farabi*. Madrid: Consejo Superior de Investigaciones Científicas.
- Augustine of Hippo. 1845. *Patrologiae Cursus Completus Tomus XXXII. S. Aurelii Augustini Tomus Primus*. Paris: J. P. Migne.
- Augustine of Hippo. 1864. *Patrologiae Cursus Completus Tomus XLI. S. Aurelii Augustini Tomus Septimus*. Paris: J. P. Migne.
- Augustine of Hippo. 1865a. *Patrologiae Cursus Completus Tomus XXXIV. S. Aurelii Augustini Tomus Tertiusm Pars Prior*. Paris: J. P. Migne.
- Augustine of Hippo. 1865b. *Patrologiae Cursus Completus Tomus XXXVI. S. Aurelii Augustini Tomus Quartus Prior*. Paris: J. P. Migne.
- Augustine of Hippo. 1865c. *Patrologiae Cursus Completus Tomus XXXVI. S. Aurelii Augustini Tomus Quartus Pars altera*. Paris: J. P. Migne.
- Augustine of Hippo. 1957. *Obras de San Agustín XV. De la doctrina cristiana. Del Génesis contra los maniqueos. Del Génesis a la letra, incompleto. Del Génesis a la letra*. Madrid: La Editorial Católica, S.A.

- Augustine of Hippo. 1958a. *Obras de San Agustín II Las Confesiones*, 8th ed. Madrid: La Editorial Católica, S.A.
- Augustine of Hippo. 1958b. *Obras de San Agustín XVI La Ciudad de Dios*, 8th ed. Madrid: La Editorial Católica, S.A.
- Augustine of Hippo. 1958c. *Obras de San Agustín XVII La Ciudad de Dios, Libros XIII-XX*, 8th ed. Madrid: La Editorial Católica, S.A.
- Augustine of Hippo. 1962. *Obras de San Agustín III. Obras Filosóficas*, 3rd ed. Madrid: La Editorial Católica, S.A.
- Augustine of Hippo. 1964. *Obras de San Agustín XIX. Enarraciones sobre los Salmos (1)*. Madrid: La Editorial Católica, S.A.
- Augustine of Hippo. 1967. *Obras de San Agustín XXII. Enarraciones sobre los Salmos (4)*. Madrid: La Editorial Católica, S.A.
- Augustine of Hippo. 1969. *Obras de San Agustín I. Introducción general y primeros escritos*, 4th ed. Madrid: La Editorial Católica, S.A.
- Augustine of Hippo. 1988. *Obras de San Agustín XXXII. Escritos varios (1). La inmortalidad del alma. La música. La fe y el símbolo de los apóstoles. La catequesis a principiantes. La fe y las obras. Sermón sobre la disciplina cristiana.-Sermón a los catecúmenos sobre el símbolo de los apóstoles*. Madrid: La Editorial Católica, S.A.
- Baiges, Ignasi J. Baiges. 1999. Els manuscrits de la Catedral de Tortosa en un inventari de 1420. *Anuario De Estudios Medievales* 29: 3–20. [\[CrossRef\]](#)
- Bayerni, Enrique. 1962. *Los Códices Medievales de la Catedral de Tortosa. Novísimo Inventario Descriptivo*. Tortosa: Talleres Gráficos Algueró y Baiges.
- Bovelles, Carolus. 1510. *Liber de intellectu, Liber de sensibus, Libellus de Nihilo, Ars oppositorum, Liber de generatione. Liber de Sapiente, Liber de duodecim numeris*. Paris: Henri Estienne.
- Bovelles, Carolus. 1542. *Livre singulier et utile, touchant l'art pratique de geometrie, compose nouvellement en françoys, par maistre Charles de Bouvelles*. Paris: Regnaud Chaudière et Claude.
- Bubnov, Nicolaus. 1899. *Gerberti postea Silvestri II papae opera mathematica (972–1003)*. Berlin: Friedländer.
- Calvo, José Antonio. 2014. *Los cabildos hispánicos de canónigos regulares de la obediencia de San Rufo de Avignon (Siglos XI–XV)*. Sevilla: Universidad de Sevilla.
- Cilleruelo, Lope. 1953. Origen del simbolismo del número siete en san Agustín. *La Ciudad de Dios* 165: 501–11.
- Denifle, Heinrich, and Emile Chatelain. 1896. *Inventarium Codicum Manuscriptorum Capituli Dertusensis*. Paris: Apud Aemilium Bouillon, Editorem.
- Dürer, Albrecht. 1525. *Underweysung der Messung, mit dem Zirckel und Richtscheyt: In Linien Ebenen vo gantzen Corporen*. Nürnberg: Hieronymum Formschneyder.
- Font i Carreras, August. 1891. *La catedral de Barcelona: Ligeras consideraciones sobre su belleza arquitectónica*. Barcelona: Asociación de Arquitectos de Cataluña.
- García, M. 1998. *La catedral de Tortosa. Aproximación a su ser y a su quehacer*. Tortosa: Cooperativa Gráfica Dertosense.
- Gauss, Carl Friedrich. 1801. *Disquisitiones Arithmeticae Auctore d. Carolo Federico Gauss*. Lipsiae: In Commissis apud Gerh. Fleischer Jun.
- González, Ángel. 1932. *Alfarabi. Catálogo de las ciencias*. Madrid: Facultad de Filosofía y Letras.
- Grimaldi, Matilde. 2015. La ilustración de manuscritos en Tarragona (siglos IX al XIII): El Scriptorium de Santes Creus, y los fondos del Arxiu Històric Arxidiocesà de Tarragona y de la Biblioteca del Arxiu Capitular de la Catedral de Tortosa. In *Tarragona: Enciclopedia del románico en Cataluña*. Palencia: Fundación Santa María la Real Centro de Estudios del Románico, pp. 77–90.
- Guitarte, Vidal. 1987. Incunables de la biblioteca catedralicia de Tortosa. *Anales Valentinus* 8: 378–90.
- Heath, Thomas Little. 1908. *The Thirteen Books of Euclid's Elements*. 3 vols. Cambridge: Cambridge University Press.
- Heideloff, C. A. 1844. *Die Bauhütte des Mittelalters in kurzgefasste geschichtliche Darstellung mit Urkunden und andern Beilagen, so wie einer Abhandlung über den Spitzbogen in der Architektur der Alten*. Nuremberg: Berlag von Johann Adam Stein.
- Ibarburu, María Eugenia. 1984. Estudio iconográfico de la “Ciudad de Dios” de San Agustín, código 20. *D' Art: Revista del Departament d'Historia de l'Arte* 10: 93–124.
- Ibarburu, María Eugenia. 1985. Algunos comentarios estilísticos sobre la Ciudad de Dios de San Agustín, Código 20 del Archivo Capitular de Tortosa. *D' Art* 11: 103–21.
- Isidoro de Sevilla. 1862. *Patrologiae Cursus Completus Tomus LXXXII. Ssncti Isidori Hispalensis Episcopi, Opera Omnia. Tomus Quintus*. Paris: J.-P. Migne.
- Kepler, Johannes. 1619. *Ioannis Keppleri Harmonices mundi libri V: Quorum primus Geometricus, de figurarum regula-rium, quae proportionibus harmonicis constituunt, ortu y demonstrationibus. ... Lincii Austriae: Sumptibus Go-dofredi Tampachii bibl. Francof., Excudebat Ioannes Plancus*.
- Laborde, A. 1909. *Les manuscrits à peintures de la Cité de Dieu de Saint Augustin. 3 Tomes*. Paris: Pour la Société des bibliophiles français, É. Rahir.
- Lluis i Ginovart, Josep, and Cinta Lluis-Teruel. 2023. Incommensurability, the Heptagon of Ignacio Muñoz (1684), and Kepler's Heresy. *Math Intelligencer* 45: 108–17. [\[CrossRef\]](#)
- Lluis i Ginovart, Josep, Gerard Fortuny Anguera, Agustí Costa Jover, and Pau de Sola-Morales Serra. 2013. Gothic construction and the traça of a heptagonal apse. The problem of the heptagon. *Nexus Network Journal: Architecture and Mathematics* 15: 325–48. [\[CrossRef\]](#)
- López, Ignacio. 2016. Uti y frui: La doble dimensión del amor en el pensamiento de Agustín de Hipona. *Cuadernos de Teología* 8: 104–25. [\[CrossRef\]](#)
- Mâle, Émile. 1910. *L'art religieux du XIIIe siècle en France: étude sur l'iconographique du Moyen Age et sur ses sources d'inspiration*. Paris: Armand Colin Éditeurs.
- March, José María. 1916. Miniaturas españolas antiguas. Biblioteca Capitular de Tortosa, Código núm. 20. *Razón y Fe* 45: 351–54.

- Misonne, Daniel. 1963. La législation canoniale de Saint-Ruf d'Avignon à ses origines. Règle de Saint Augustin et coutumier. *Annales du Midi* 75: 471–89. [\[CrossRef\]](#)
- O'Callaghan, Ramon. 1887. *Anales de Tortosa. Tomo II*. Tortosa: Imprenta Católica de Gabriel Llasat.
- Orozco, Alonso. 1881. *La regla de nuestro gran padre y patriarca San Agustín en latín y castellano, para uso manual de los que la profesan, con una breve declaración de algunos lugares que la necesitan por el R.P. Fr. Alonso de Orozco*. Manila: Imp. de los Amigos del País.
- Panofsky, Erwin. 1951. *Gothic Architecture and Scholasticism*. Pennsylvania: Archabbey Press.
- Pardillos, María Teresa. 2000. El libro de los números atribuido a San Isidoro, Obispo de Sevilla. *Emblemata* 6: 285–305.
- Pola, Víctor Manuel. 2003. Una devoción para un pueblo: María en el camino de los taustanos: Historia del culto a la Virgen en la Villa de Tauste. In *Actas de las V Jornadas Sobre la Historia de Tauste 1 al 5 de diciembre de 2003*. Tauste: 3 Asociación Cultural "El Patiaz", pp. 9–88.
- Puigarnau, Alfons. 2023. Ernst Kantorowicz's *Synthronos*: New Perspectives on Medieval Charisma. *Religions* 14: 914. [\[CrossRef\]](#)
- Puig i Cadafalch, José. 1923. *El problema de la transformació de la catedral del nord importada a Catalunya: Contri-bució a l'estudi de l'arquitectura gòtica meridional. Miscel·lània Partas de la Riba*. Barcelona: Institut d'Estudis Catalans.
- Raynaud, Dominique. 2012. Abū al-Wafā' Latinus? A study of method. *Historia Mathematica* 39: 34–83. [\[CrossRef\]](#)
- Roritzer, M. 1999. *Das Büchlein von der Fialen Gerechtigkeit (fak. Regensburg 1486) und Die Geometria Deutsch (fak. Regensburg un 1487/88)*. Regensburg: Hürtgenwald Guido Pressler.
- Rubió, A., and J. Rubió. 1914. La Biblioteca del Capítol Catedral de Tortosa. In *Anuari de l'Institut d'Estudis Catalans, (1913–1914)*. Barcelona: Institut Estudis Catalans, pp. 745–57.
- Simson, Otto G. 1952. The Gothic Cathedral: Design and Meaning. *Journal of the Society of Architectural Historians* 11: 6–16. [\[CrossRef\]](#)
- Simson, Otto G. 1956. *The Gothic Cathedral: The Origins of Gothic Architecture and the Medieval Concept of Order*. New York and Evanston: Harper&Row.
- Street, George Edmund. 1865. *Some Account of Gothic Architecture in Spain*. London: John Murray.
- Toomer, Gerald J. 1984. *Ptolemy's Almagest*. Translated and Annotated by Gerald J. Toomer. With a Foreword by Owen Gingerich. London: Duckworth.
- Trottmann, Christian. 2015. Science, sagesse et jouissance, d'Augustin à Charles de Bovelles. *Quaestio* 15: 805–16. [\[CrossRef\]](#)
- Voragine, J. 1999. *Santiago de la Vorágine. La leyenda dorada, 1. Traducción del latín: Fray José Manuel Macías*. Madrid: Alianza Editorial, S. A.
- Worringer, Wilhelm. 1911. *Formprobleme der Gotik*. München: Piper Verlag.

**Disclaimer/Publisher's Note:** The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.